

# MATHEMATICS EXAM REQUIREMENTS

## I. Real Numbers. The student:

- 1) performs operations (addition, subtraction, multiplication, division, exponentiation, root extraction, logarithms) on the set of real numbers;
- 2) performs simple proofs concerning the divisibility of integers and remainders of division, e.g.:
  - a) a proof that the product of four consecutive integers is divisible by 24,
  - b) a proof of the property: if a number when divided by 4 gives a remainder of 3, then it is not a square of an integer;
- 3) applies the properties of roots of any degree, including odd roots of negative numbers;
- 4) applies the relationship between square root and exponentiation and the laws of operations on powers and square roots;
- 5) applies the monotonicity of exponentiation, in particular the properties: if  $x < y$  and  $a > 1$ , then  $a^x < a^y$ , and if  $x < y$  and  $0 < a < 1$ , then  $a^x > a^y$ ;
- 6) uses the concept of a numerical interval, marking intervals on a number line;
- 7) applies the geometric and algebraic interpretation of absolute value, solves equations such as:  $|x + 4| = 5$ ;
- 8) uses the properties of exponentiation and square root in practical situations, including calculating compound interest, deposit yields, and loan costs;
- 9) applies the relationship between logarithms and exponentiation, and uses the formulas for the logarithm of a product, logarithm of a quotient, and logarithm of a power.

## II. Algebraic expressions. The student:

- 1) applies the short multiplication formulas for:  $(a + b)^2$ ,  $(a - b)^2$ ,  $a^2 - b^2$ ;
- 2) adds, subtracts, and multiplies polynomials of one and more variables;
- 3) excludes a monomial from an algebraic sum;
- 4) multiplies and divides rational expressions.

## III. Equations and inequalities. The student:

- 1) transforms equations and inequalities in an equivalent manner, including, for example, transforms the equation  $\frac{5}{x+1} = \frac{x+3}{2x-1}$  to the equivalent form;
- 2) interprets identity and inconsistent equations and inequalities;
- 3) solves linear inequalities with one unknown;
- 4) solves quadratic equations and inequalities;
- 5) solves polynomial equations  $W(x) = 0$  for polynomials written in factored form.

## IV. Systems of equations. The student:

- 1) solves systems of linear equations with two unknowns, provides a geometric interpretation of consistent and inconsistent systems;
- 2) uses systems of equations to solve problems.

## V. Functions. The student:

- 1) defines functions as unambiguous assignments using a verbal description, table, graph, formula (including different formulas on different intervals);
- 2) calculates the value of a function given by an algebraic formula;
- 3) reads and interprets the values of functions defined using tables, graphs, formulas etc., even when using the same source of information multiple times or several sources simultaneously;

- 4) reads from the graph of a function: domain, range, zeros, intervals of monotonicity, intervals in which the function takes values greater (not less) or less (not greater) than a given number, the largest and smallest values of the function (if they exist) in a given closed interval, and arguments for which the function takes the largest and smallest values;
- 5) interprets the coefficients in the formula of a linear function;
- 6) determines the formula of a linear function based on information about its graph or its properties;
- 7) sketches the graph of a quadratic function given by the formula;
- 8) interprets the coefficients in the formula of a quadratic function in general, vertex, and factored forms (if it exists);
- 9) determines the formula of a quadratic function based on information about the function or its graph;
- 10) determines the largest and smallest value of a quadratic function in a closed interval;
- 11) uses the properties of linear and quadratic functions to interpret geometric, physical, and other problems, including those in a practical context;
- 12) uses the graph of the function  $y = f(x)$  to sketch the graphs of the function  $y = f(x - a)$ ,  $y = f(x) + b$ ;
- 13) uses the function  $f(x) = \frac{a}{x}$ , including its graph, to describe and interpret problems related to inversely proportional quantities, including practical applications;
- 14) uses exponential and logarithmic functions, including their graphs, to describe and interpret problems related to practical applications.

VI. Sequences. The student:

- 1) calculates the terms of a sequence defined by a general formula;
- 2) calculates the initial terms of sequences defined recursively;
- 3) in simple cases, examines whether a sequence is increasing or decreasing;
- 4) checks whether a given sequence is arithmetic or geometric;
- 5) applies the formula for the  $n$ -th term and the sum of the first  $n$  terms of an arithmetic sequence;
- 6) applies the formula for the  $n$ -th term and the sum of the first  $n$  terms of a geometric sequence;
- 7) uses the properties of sequences, including arithmetic and geometric, to solve problems, including those set in a practical context.

VII. Trigonometry. The student:

- 1) uses the definitions of functions: sine, cosine, and tangent for angles from  $0^\circ$  to  $180^\circ$ , in particular, determines the values of trigonometric functions for angles  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$ ;
- 2) uses the formulas  $\sin^2 \alpha + \cos^2 \alpha = 1$ ,  $\operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha}$ ;
- 3) uses the law of cosines and the formula for the area of a triangle  $P = \frac{1}{2} \cdot a \cdot b \cdot \sin \gamma$ ;
- 4) calculates the angles of a right triangle and the lengths of its sides given the appropriate data (solves right triangles, including using trigonometric functions).

VIII. Plane geometry. The student:

- 1) determines the radii and diameters of circles, the lengths of chords of circles, and tangent segments, including using the Pythagorean Theorem;
- 2) recognizes acute-angled, right-angled, and obtuse-angled triangles given the side lengths (including using the converse of the Pythagorean Theorem and the law of cosines); applies the theorem: in a triangle, the longer side lies opposite the larger interior angle;
- 3) recognizes regular polygons and uses their basic properties;
- 4) uses the properties of angles and diagonals in rectangles, parallelograms, rhombuses, and trapezoids;
- 5) uses the properties of inscribed and central angles;

- 6) uses the formulas for the area of a sector of a circle and the length of an arc of a circle;
- 7) uses Thales' theorem;
- 8) uses the properties of similar triangles;
- 9) uses the relationships between the perimeters and areas of similar figures;
- 10) identifies the basic points of interest in a triangle: the center of a circle inscribed in the triangle, the center of the circle circumscribing a triangle, the orthocenter, and the center of gravity, and uses their properties;
- 11) performs geometric proofs;
- 12) uses trigonometric functions to determine the lengths of segments in plane figures and calculate the areas of figures.

IX. Analytic geometry on the Cartesian plane. The student:

- 1) recognizes the mutual position of lines in a plane based on their equations, including finding the common point of two lines, if one exists;
- 2) uses equations of lines in a plane, in slope and general form, including determining the equation of a line with given properties (such as, for example, passing through two given points, a known slope, parallelism to another line);
- 3) calculates the distance between two points in a coordinate system;
- 4) uses the circle equation  $(x - a)^2 + (y - b)^2 = r^2$ ;
- 5) determines images of circles and polygons with axial symmetries about the axes of the coordinate system, and central symmetry (centered at the origin).

X. Stereometry. The student:

- 1) recognizes the mutual position of lines in space, especially non-intersecting perpendicular lines;
- 2) uses the concept of an angle between a line and a plane and the concept of a dihedral angle between half-planes;
- 3) recognizes angles between segments (e.g., edges, and diagonals) and angles between faces in prisms and pyramids, and calculates the measures of these angles;
- 4) recognizes the angle between segments and angles between segments and planes in cylinders and cones (e.g., the angle of inclination of a cone, the angle between a generatrix and a base), and calculates the measures of these angles;
- 5) calculates the volumes and surface areas of prisms, pyramids, cylinders, cones, and spheres, also using trigonometry;
- 6) uses the relationship between the volumes of similar solids.

XI. Combinatorics. The student:

- 1) counts objects in simple combinatorial situation;
- 2) counts objects using the rules of multiplication and addition (also in combinaton) for any number of operations, e.g.:
  - a) counting the number of four-digit odd positive integers such that their decimal representation contains exactly one digit 1 and exactly one digit 2,
  - b) counting the number of four-digit even positive integers such that their decimal representation contains exactly one digit 0 and exactly one digit 1.

XII. Probability and Statistics. The student:

- 1) calculates probability in the classical model;
- 2) calculates the arithmetic mean and weighted mean, and finds the median and mode.

XIII. Optimization. The student solves optimization problems in situations that can be described by a quadratic function.